Exercise 7

Find dy/dx by implicit differentiation.

$$x^4 + x^2y^2 + y^3 = 5$$

Solution

Differentiate both sides with respect to x.

$$\frac{d}{dx}(5) = \frac{d}{dx}(x^4 + x^2y^2 + y^3)$$

$$0 = \frac{d}{dx}(x^4) + \frac{d}{dx}(x^2y^2) + \frac{d}{dx}(y^3)$$

$$= (4x^3) + \left[\frac{d}{dx}(x^2)\right]y^2 + x^2\left[\frac{d}{dx}(y^2)\right] + \left[3y^2 \cdot \frac{d}{dx}(y)\right]$$

$$= 4x^3 + (2x)y^2 + x^2\left[2y \cdot \frac{d}{dx}(y)\right] + (3y^2y')$$

$$= 4x^3 + 2xy^2 + 2x^2yy' + 3y^2y'$$

Solve for y'.

$$y' = -\frac{4x^3 + 2xy^2}{2x^2y + 3y^2}$$